Advanced Mathematics

Upper Secondary

Teacher Guide

Papua New Guinea
Department of Education

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Contents

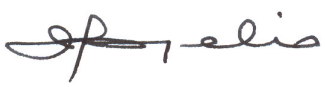
Secretary's message

This teacher guide is to be used by teachers when implementing the Upper Secondary Advanced Mathematics Syllabus (Grades 11 and 12) throughout Papua New Guinea. The Advanced Mathematics syllabus identifies the learning outcomes and content of the subject as well as assessment requirements. The teacher guide gives practical ideas about ways of implementing the syllabus: suggestions about what to teach, strategies for facilitating learning and teaching, how to assess and suggested assessment tasks.

A variety of suggested learning and teaching activities provides teachers with ideas to motivate students to learn, and make learning relevant, interesting and enjoyable. Teachers should relate learning in Advanced Mathematics to real contexts and issues and the local environment. Teaching using meaningful contexts and ensuring students participate in appropriate practical activities assists students to gain knowledge and understanding, and demonstrate skills in Advanced Mathematics.

Teachers are encouraged to integrate Advanced Mathematics activities with other subjects, where appropriate, so that students can see the interrelationships between subjects and that the course they are studying provides a holistic education and a pathway for the future.

I commend and approve the Advanced Mathematics Teacher Guide for use in all schools with Grades 11 and 12 students throughout Papua New Guinea.



DR JOSEPH PAGELIO

Secretary for Education

Introduction

The purpose of this teacher guide is to help you to implement the Advanced Mathematics syllabus. It is designed to support and assist you in planning your teaching strategies and learning activities and assessment tasks. It also encourages you to develop activities that are appropriate and relevant.

The teacher guide supports the syllabus. The syllabus states the learning outcomes for the subject and units, and outlines the content and skills that students will learn, and the assessment requirements.

The teacher guide provides direction for you in using the outcomes approach in your classroom. The outcomes approach requires you to consider the assessment requirements early in your planning. This is reflected in the teacher guide.

This teacher guide provides examples of learning and teaching strategies. It also provides detailed information on criterion-referenced assessment, and the resources needed to teach Advanced Mathematics. The section on recording and reporting shows you how to record students' marks and how to report against the learning outcomes.

The teacher guide is intended to guide teachers to implement the syllabus.

The outcomes approach

In Papua New Guinea, the Lower Secondary and Upper Secondary syllabuses use an outcomes approach. The major change in the curriculum is the shift to what students know and can do at the end of a learning period, rather than a focus on what the teacher intends to teach.

An outcomes approach identifies the knowledge, skills, attitudes and values that all students should achieve or demonstrate at a particular grade in a particular subject (the learning outcomes). The teacher is responsible for identifying, selecting and using the most appropriate teaching methods and resources to achieve these learning outcomes.

Imagine the student is on a learning journey, heading to a destination. The destination is the learning outcome that is described in the syllabus document. The learning experiences leading to the learning outcomes are to be determined by the teacher. The teacher uses curriculum materials, such as syllabus documents and teacher guides, as well as textbooks or electronic media and assessment guidelines to plan activities that will assist students achieve the learning outcomes. The outcomes approach has two purposes. They are:

- to equip all students with knowledge, understandings, skills, attitudes and values needed for future success
- to implement programs and opportunities that maximise learning.

Three assumptions of outcomes-based education are that:

- all students can learn and succeed (but not on the same day or in the same way)
- success breeds further success
- schools can make a difference.

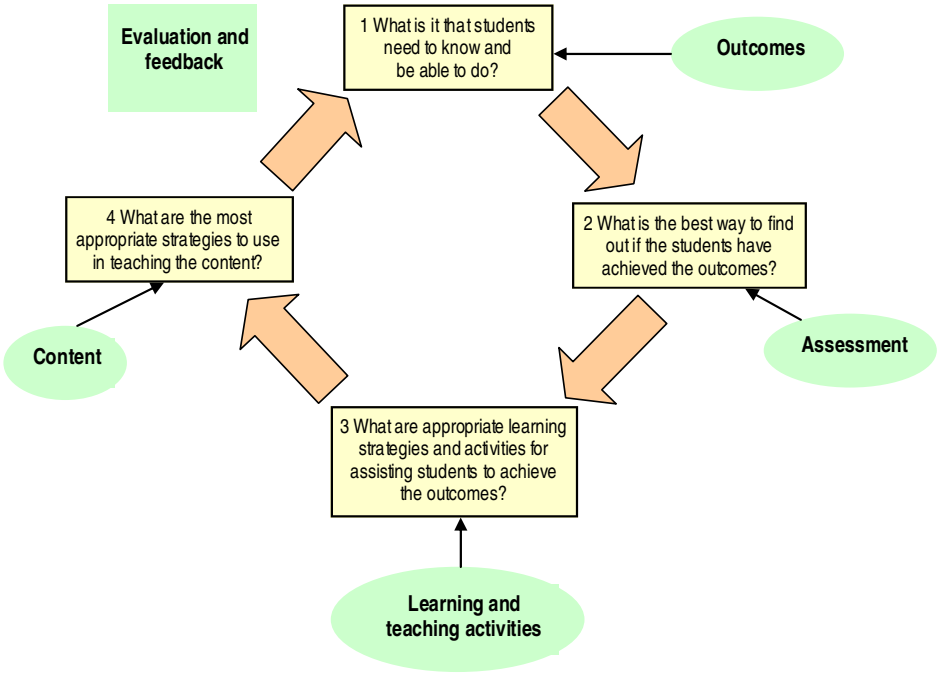
The four principles of the outcomes approach in Papua New Guinea are:

1. *Clarity of focus through learning outcomes*
   This means that everything teachers do must be clearly focused on what they want students to be able to do successfully. For this to happen, the learning outcomes should be clearly expressed. If students are expected to learn something, teachers must tell them what it is, and create appropriate opportunities for them to learn it and demonstrate their learning.
2. *High expectations of all students*
   This means that teachers reject comparative forms of assessment and embrace criterion-referenced approaches. The principle of high expectations is about insisting that work be at a very high standard before it is accepted as completed, while giving students the time and support they need to reach this standard. At the same time students begin to realise that they are capable of far more than before and this challenges them to aim even higher.
3. *Expanded opportunities to learn*
   This is based on the idea that not all students can learn the same thing in the same way in the same time. Some achieve the learning outcomes sooner and others later. However, most students can achieve high standards if they are given appropriate opportunities. Traditional ways of

organising schools do not make it easy for teachers to provide expanded opportunities for all students.

4. *Planning and programming by designing down*
   This means that the starting point for planning, programming and assessing must be the learning outcomes—the desired end results. All decisions on inputs and outputs are then traced back from the learning outcomes. The achievement of the outcome is demonstrated by the skills, knowledge and attitudes gained by the student. The syllabuses and/or teacher guides describe some ways in which students can demonstrate the achievement of learning outcomes.

Outcomes-based approach



Learning outcomes provide teachers with a much clearer focus on what students should learn. They also give teachers greater flexibility to decide what is the most appropriate way of achieving the learning outcomes and meeting the needs of their students by developing programs to suit local content and involve the community.

The outcomes approach promotes greater accountability in terms of student achievement because the learning outcomes for each grade are public knowledge—available to teachers, students, parents and the community. It is not the hours of instruction, buildings, equipment or support services that are the most important aspect of the education process but rather, what students know and can do, as they progress through each grade. The outcomes approach means that learning

- has a clearer purpose
- is more interactive—between teacher and students, between students
- has a greater local context than before
- is more closely monitored and acted upon by the teacher
- uses the teacher as a facilitator of learning as well as an imparter of knowledge.

Learning outcomes

The Advanced Mathematics learning outcomes describe what students know and can do at the end of Grade 12. The level of achievement of the learning outcome should improve during the two years of Upper Secondary study, and it is at the end of the study that students are given a summative assessment on the level of achievement of the learning outcome.

Students can:

1. communicate mathematical processes and results verbally and in writing
2. measure and use appropriate techniques and instruments to estimate and calculate physical quantities
3. apply knowledge of numbers and their relationships to investigate a range of different contexts
4. identify, interpret, describe and represent various functional relationships to solve problems in real and simulated contexts
5. interpret, describe and represent properties of and relationships between 2-dimensional shapes and 3-dimensional objects in a variety of orientations and positions
6. apply knowledge of statistics and probability to communicate, justify, predict and critically analyse findings and draw conclusions
7. describe and explain interrelationships between mathematical concepts
8. analyse mathematical situations and construct and apply logical arguments in resolution of mathematical problems
9. plan, organise and carry out mathematical activities individually and cooperatively.

Learning and teaching

You, as a teacher, must teach the knowledge that is included in the syllabus documents. Not only do you have to be able to teach what students should know, you must also be able to interpret that knowledge for students in a way that makes it relevant to them, and enables them to begin to acquire skills of analysis and problem solving, which support learning and teaching. You also need to give students some opportunities to apply their knowledge, to be creative and to solve problems.

Learning and teaching strategies

Students who participate in guided instruction learn more than students who are left to construct their own knowledge (Mayer 2004). You need to employ a variety of learning and teaching approaches because all students do not learn in the same way. The auditory learner prefers to use listening as the main way of learning new material whereas a visual learner prefers to see things written down. Students should be actively involved in their learning, so you need to design appropriate practical activities or experiments using resources that can be found in your location.

In Grades 11 and 12, students will already have had a wide variety of experiences. You need to make use of your students' experiences when designing and conducting learning in class; learning that is connected to your students' world.

To assist and encourage students to learn, you perform certain tasks. These are referred to as 'teaching strategies'. You need to engage students directly in learning, but there are times when you have to take charge of the learning in the class and teach particular concepts or ideas. There are many learning and teaching strategies described in the Lower Secondary teacher guides. Teaching strategies include:

- project work
- group work and cooperative learning
- classroom displays
- models
- learning games and role-play
- mind maps or concept maps
- reflective learning
- task cards.

The most efficient and long-lasting learning occurs when teachers encourage the development of higher-order thinking and critical analysis skills, which include applying, analysing, evaluation and creating. Attention should also be paid to developing students' affective and psychomotor skills. To ensure that this occurs, you should encourage deep or rich, rather than shallow, coverage of knowledge and understandings.

Developing Advanced Mathematics skills

Students need to develop mathematical skills and techniques. Skills development should happen as a part of students' learning experiences and the learning and practising of skills needs to take place in the context of Advanced Mathematics. Skills learning tends to be most effective when:

- students go from the known to the unknown
- students understand why it is necessary to master specific skills
- skills are developed sequentially at increasing levels of difficulty
- students identify the components of the skill
- the whole skill and the components of the skills are demonstrated
- there are frequent opportunities for practice and immediate feedback
- the skills being taught are varied in terms of amount and type, according to the needs of students
- the skill is used in a range of contexts.

What do students do in Advanced Mathematics?

Problem-solving strategies

Problem solving enables students to:

- identify problems
- apply strategies
- use concepts, procedures, and tools.

Representation

- charts
- graphs

Reasoning

- interpret
- generalise

Communication

- clear
- organised
- complete
- detailed
- mathematical (language, terminology, symbols, notations)

Research

Research is an essential activity in the study of Advanced Mathematics. It allows students to search for and gather information either within or outside the classroom. It supplies students with past and present mathematical information. Research enables students to gather information about Advanced Mathematics through reading and fieldwork.

Listening to guest speakers

Listening to guest speakers is an invaluable learning activity in the study of Advanced Mathematics. This activity updates students on current mathematical issues, information and practices. It encourages students to develop a critical and enquiring mind. Listening to guest speakers will enable students to:

- acquire updated knowledge about ratio, rates, percentages and measurement from those in their community or nearby public or private institutions
- listen, analyse, synthesise and interpret disseminated information.

Awareness

Awareness is a useful activity in that it educates students about new information on hazards and or benefits. Students are better equipped to in turn carry out their own awareness. Awareness enables students to:

- acquire knowledge and bring awareness to the community
- select, organise and communicate information on HIV and AIDS and how it has a ripple effect on the family, community and the nation.

Use of technology

- calculators
- computers
- computer software
- the internet

Multimedia presentations

Multimedia is a tool that can be used to explain abstract mathematical processes, concepts and systems. It enhances learning and teaching skills through audio, visual colour and motion in a variety of ways. Multimedia presentations enable students to:

- acquire knowledge about Advanced Mathematics through video, slide and print media
- expand knowledge through the use of information and communication technology.

Developing a program

A teaching program outlines the nature and sequence of learning and teaching necessary for students to demonstrate the achievement of the learning outcomes. The content of the syllabus describes the learning context and the knowledge required for the demonstration of each outcome. The relevant learning outcomes for each unit or topic are stated at the beginning of the unit and the requirements of the outcomes are elaborated.

Teachers must develop programs that include appropriate learning activities to enable students to develop the knowledge and skills identified in outcome statements. The content prescribed in the units indicates the breadth and depth with which topics should be treated. The sequence of teaching is

prescribed by the sequence of content. The learning outcomes and assessment, however, must be central to planning the teaching program.

Planning and programming units

The main purpose of planning and programming is to help you to arrange the presentation of the unit in an organised manner. This will help you to know what to teach and when to teach it. It is strongly recommended that you make plans with the other teachers who teach the same subject. By planning together, you will *all* have better lessons and make better use of your limited resources.

Points to consider when programming

- Which outcomes are students working towards?
- What is the purpose of this unit or topic or learning experience?
- Which learning experiences will assist students to develop their knowledge and understandings, skills, and values and attitudes in the subject?
- What indicators of student learning that would you expect to observe?
- How can the learning experiences be sequenced?
- How do the learning experiences in the unit relate to students' existing knowledge and skills?
- How are individual learning needs to be catered for?
- What are the literacy demands of this unit or learning experience?
- What authentic links can be made with the content of other subjects?
- How can school events and practices be incorporated into the program?
- Do the assessment methods address the outcomes and enhance the learning?
- How can the assessment be part of the learning and teaching program?

The planning process

In this teacher guide, ideas for learning and teaching activities and assessment tasks have been provided to help you teach the units. To plan a unit, these steps follow the thinking processes involved in the outcomes approach.

Step 1: Interpreting the learning outcomes

The first step is to read the description in the syllabus and then study the learning outcomes and what students do to achieve the learning outcome, to determine what students will know and be able to do by the end of the unit. You need to look at the action verb, concept and context of each learning outcome. This will help you see what skills and knowledge are embedded in the outcome.

Step 2: Planning for assessment

It is necessary to plan for assessment early to ensure that you teach the content and skills students need to achieve the learning outcomes. You will have to decide when to schedule assessment tasks to allow yourself time to teach the required content and time for students to develop the necessary skills. You will also need time to mark the task and provide feedback.

Practical tasks may, for example, be broken into a series of stages that are marked over several weeks as students progress with making their product. It is not appropriate to leave all assessment until the end of the unit.

This teacher guide provides performance standards and examples of a marking guide. You should develop marking guides when you are marking tasks to ensure consistency in your assessment. You must also develop clear and detailed instructions for completing the task and make sure all students know exactly what they have to do.

Step 3: Programming a learning sequence

This step requires you to develop a program outlining a sequence of topics and the amount of time spent on each topic. If the unit involves a project, for example, you may plan to teach some theory at appropriate stages during the project, rather than teaching all theory before students start the project.

To develop your program you need to study the topics listed in the syllabus and to think about the learning activities that will best provide students with the opportunity to learn the content and practise the appropriate skills, and how long the activities will take. You will have to think about some major activities that last several weeks and smaller activities that may be completed in a single lesson.

Step 4: Elaboration of activities and content

Once you have mapped out your program for the term, you must then develop more detailed plans for each topic in the unit. All units require students to be actively engaged in learning, not just copying from the board. Make sure you develop a range of activities that suit all learning needs—some reading and writing, some speaking and listening, some observing and doing. Browse through the textbooks and teaching resources you have access to and list chapters, pages or items that you will use for each topic in your program. The textbooks should also provide you with ideas for activities related to the topic. You may have to collect or develop some resources for yourself. Once you have sorted out your ideas and information, you can then develop your more detailed weekly program and daily lesson plans.

This teacher guide gives some suggested learning and teaching activities for each unit and some suggested assessment tasks, which you might like to use to ensure active learning.

Using the internet for classroom activities

Planning

- Where appropriate, incorporate computer sessions as part of planned learning experiences.
- Be aware that computers can be time-consuming and may require extra teacher support at unexpected times.
- Consider methods of troubleshooting, such as having students with computer expertise designated as computer assistants.
- Design activities that provide the opportunity for students to access, compare and evaluate information from different sources.
- Check protocols, procedures and policies of your school and system regarding the use of the internet.

Managing

- Ensure that all students have the opportunity to explore and familiarise themselves with the technologies, navigation tools, e-mail facilities and texts on the internet. It is likely that students have varying degrees of expertise in searching for information and navigating the internet. Students also have varying experiences and familiarity with the way texts are presented on the World Wide Web.
- Ensure that all students have an understanding of how to access the internet and how to perform basic functions, such as searching, sending and receiving e-mail.
- Students with more experience in using the internet may have information that will benefit the whole class. Provide opportunities for students to share their experiences, interests, information and understandings. As well as planning lessons to instruct students in these skills, pairing students and peer tutoring on the computer can enable more experienced students to assist other students.
- Ensure that students critically analyse mathematical information gathered on the internet just as they would for any other text. They should be aware that material posted on the Web is not necessarily subject to the conventional editorial checks and processes generally applied to print-based publications. When evaluating information, students might consider:
  - the intended audience of the site
  - bias in the presentation of information, or in the information itself, including commercial or political motives
  - accuracy of information
  - balanced points of view
  - currency of information, including publishing dates
  - authority of source or author (institution, private individual)
  - ownership of the website (such as corporate, small business, government authority, academic
  - cultural or gender stereotyping.
- Ensure that software and hardware (computer, modem) are maintained in good working order.
- Ensure that all students are given equal opportunities to use the computer.

Assessing student work containing material from the internet

- Students can download large quantities of information from the internet. By itself, this information provides very little evidence of student effort or student achievement. Students must make judgements about the validity and safety of information when working from the World Wide Web. They must consider the purpose of the text, identify bias and consider validity of arguments presented and the nature and quality of the evidence.
- When assessing student work that includes material drawn from the internet, it is important to recognise how students have accessed the information, what value they place on it and how they have used it for the particular unit being studied in class. It is useful to look for evidence of critical evaluation, and the development of students' capacities to access, manipulate, create, restore and retrieve information.

Advanced Mathematics requirements

There are four units in Grade 11 and three units in Grade 12, which all students must complete.

Grade	Weeks	Term	Unit	Essential resources for activities and assessment
11	10	1	Number and Application	Bathroom scale, measuring instruments (analogue or digital), mass sets, metre ruler, conversion tables, grid papers
11	12	2	Graphs and Functions	Scientific calculator, grid papers
11	10	3	Managing Data	Data from lower level government (LLG) and other government organisations, dice, deck of cards, dominoes
11	8	4	Geometry	Maths kit, trundle wheel, tape measure
12	10	1	Patterns and Algebra	Scientific calculators, grid papers
12	10	2	Trigonometry and Vectors	Tape measure, clinometer, measuring instruments (both analogue and digital, such as clocks)
12	10	3	Calculus	Grid papers, scientific calculators, pendulum, balls, string, tape measure

Assessing Advanced Mathematics

Assessment is an important part of learning and teaching. It is used to:

- evaluate and improve learning and teaching
- report achievement
- provide feedback to students on their progress
- provide feedback to stakeholders.

Criterion-referenced assessment

Assessment in Advanced Mathematics is criterion-referenced and measures students' achievement of the learning outcomes described in the syllabus. In criterion-referenced assessment, particular knowledge, skills or abilities are specified as criteria that must be achieved. The extent to which they are achieved is assessed and facilitated by the teacher.

Criterion-referenced assessment often takes on a problem-centred orientation, rather than a knowledge-based orientation. To achieve an outcome means having to demonstrate the attainment of skills and attitudes, not just write about them. Assessment becomes more than just a means of judging knowledge and performance—it becomes an integral part of the learning process itself. Criterion-referenced assessment is:

- standards or criterion-referenced; outcomes are judged against pre-defined standards (see table below)
- direct and authentic, related directly to the learning situation. This has the potential for motivating learning, since students can see a direct relevance between what is learnt and what is assessed.

Norm-referenced assessment

'Norm-referenced' assessment makes judgments on how well the student did in relation to others who took the test. It is often used in conjunction with a curve of 'normal distribution', which assumes that a few will do exceptionally well and a few will do badly and the majority will peak in the middle, normally judged as average.

Example of a criterion-referenced test

The driving test is the classic example of a criterion-referenced test. The examiner has a list of criteria, each of which must be satisfactorily demonstrated in order to pass; for example, completing a three-point turn without hitting either kerb. The important thing is that failure in one criterion cannot be compensated for by above-average performance in others; nor can a student fail in spite of meeting every criterion (as they can in norm-referenced assessment) simply because everybody else that day surpassed the criteria and was better than him or her. Criterion-referenced assessment has the following characteristics:

- a syllabus that describes what students are expected to learn in terms of aims, outcomes and content

- a syllabus that provides a clear sense of the syllabus standards through its aims, outcomes and content
- tasks designed to produce an image of what students have achieved at that point in the learning and teaching process relative to the outcomes
- standards of performance at different levels: the 'performance standards'
- a report that gives marks referenced to predetermined standards
- assessment tasks that refer to syllabus outcomes, content, assessment components and component weightings.

  external examinations that are based on syllabus outcomes and content. External markers use standards-referenced marking guidelines developed by the Mathematics Examination Committee.
- assessment that is better integrated with learning and teaching.

Criterion or standards-referenced assessment in Advanced Mathematics

Learning outcomes performance standards					
Learning outcome	**Very high achievement**	**High achievement**	**Satisfactory achievement**	**Low achievement**	**Below minimum standard**
1. Communicate mathematical processes and results verbally and in writing	Communicate complex mathematical processes and results using an extensive range of written, graphic and oral forms	Communicate mathematical information using a broad range of written, graphic and oral forms	Communicate mathematical information using a range of written, oral and graphic forms	Communicate some mathematical information using a limited range of written, oral and graphic forms	Has failed to meet the minimum standard required
2. Measure and use appropriate techniques and instruments to estimate and calculate physical quantities	Independently measures and use a range of appropriate techniques to estimate and calculate physical quantities	Measures and use appropriate techniques to estimate and calculate physical quantities	Selects and applies mathematical skills and techniques	Selects and applies limited mathematical skills and techniques	Has failed to meet the minimum standard required
3. Apply knowledge of numbers and their relationships to investigate a range of different contexts	Independently applies knowledge of numbers and their relationships to investigate a wide range of different contexts	Applies knowledge of numbers and their relationships to investigate a range of different contexts	Uses knowledge of numbers and their relationships solve mathematical problems	Investigate and solve mathematical problems with help	Has failed to meet the minimum standard required
4. Identify, interpret, describe and represent various functional relationships to solve problems in real and simulated contexts	Independently identifies, interprets, describes and represents various functional relationships to solve a range of problems in real and simulated contexts	Identifies describes and represents various functional relationships to solve problems in real and simulated contexts	Identifies describes and represents some functional relationships to solve some problems in real and simulated contexts	Identifies describes and represents a few functional relationships to solve problems in real and simulated contexts	Has failed to meet the minimum standard required

Learning outcomes performance standards					
Learning outcome	**Very high achievement**	**High achievement**	**Satisfactory achievement**	**Low achievement**	**Below minimum standard**
5. Interpret, describe and represent properties of and relationships between 2-dimensional shapes and 3-dimensional objects in a variety of orientations and positions	Gives detailed descriptions and properties of 2-dimensional shapes and 3-dimensional objects in a variety of orientations and positions	Gives some descriptions and properties of 2-dimensional shapes and 3-dimensional objects in a variety of orientations and positions	Gives a few descriptions and properties of 2-dimensional shapes and 3-dimensional objects	Gives limited descriptions and properties of 2-dimensional shapes and 3-dimensional objects	Has failed to meet the minimum standard required
6. Apply knowledge of statistics and probability to communicate, justify, predict and critically analyse findings and draw conclusions	Critically analyse, evaluate and interpret statistical dataand information	Detailed analysis, evaluation and interpretation of statistical dataand information	Satisfactory analysis, evaluation and interpretation of statistical data and information	Poor analysis, evaluation and interpretation of statistical data and information	Has failed to meet the minimum standard required
7. Describe and explain interrelationships between mathematical concepts	Demonstrates extensive knowledge and understanding of a wide range of interrelationships between mathematical concepts	Demonstrates thorough knowledge and understanding of a range of interrelationships between mathematical concepts	Demonstrates some knowledge and understanding of interrelationships between mathematical concepts	Demonstrates some knowledge of interrelationships between mathematical concepts	Has failed to meet the minimum standard required
8. Analyse mathematical situations and construct and apply logical arguments in resolution of mathematical problems	Gives logical and detailed explanations to analyse a range of mathematical situations and apply logical arguments in resolution of mathematical problems	Gives logical explanations and reasons to analyse mathematical situations and apply logical arguments in resolution of mathematical problems	Gives explanations to analyse mathematical situations	Identifies some mathematical situations	Has failed to meet the minimum standard required
9. Plan, organise and carry out mathematical activities individually and cooperatively	Demonstrate excellent planning and organising skills to carry out a wide range of mathematical activities	Demonstrate good planning and organising to carry out a range of mathematical activities	Demonstrate satisfactory planning and organising to carry out mathematical activities	Demonstrate fair planning and organising to carry out mathematical activities	Has failed to meet the minimum standard required

Assessment *for* learning

Assessment *for* learning is often called 'formative assessment'. It is assessment that gathers data and evidence about student learning during the learning process. It enables you to see where students are having problems and to give immediate feedback, which will help your students learn better. It also helps you plan your program to make student learning,

and your teaching, more effective. Often it is informal—students can mark their own work or their friend's. An example is a quick class quiz to see if students remember the important points of the previous lesson.

Assessment *of* learning

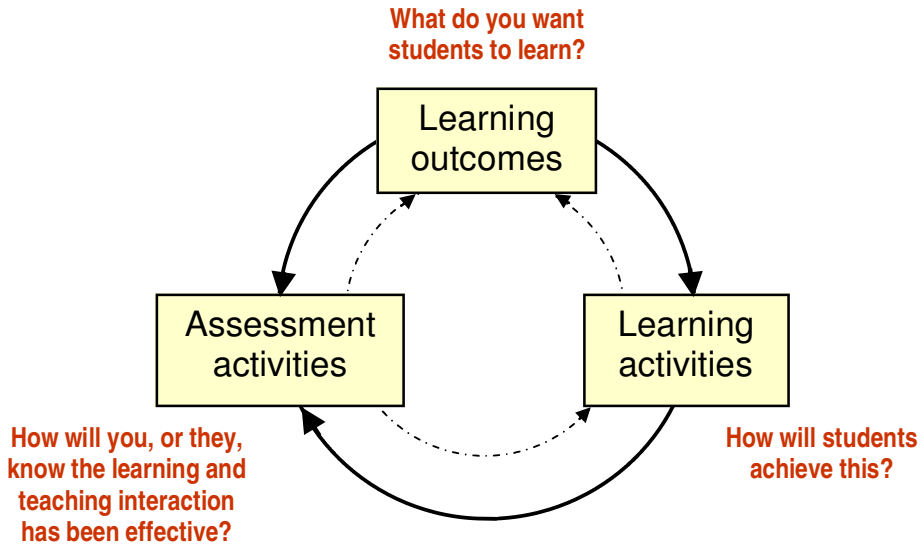
Assessment *of* learning is often called 'summative assessment'. It is used to obtain evidence and data that shows how much learning has occurred, usually at the end of the term or unit. End-of-year examinations are examples of summative assessment. It is usually done for formal recording and reporting purposes.

Assessing Advanced Mathematics units

In Advanced Mathematics the learning outcomes are assessed using the range of assessment methods specified in the syllabus. This teacher guide includes sample assessment tasks and assessment criteria, which can be used to assess the outcomes of those units. Teachers can use these samples to develop other assessment tasks, criteria and performance standards.

In deciding what to assess, the first point to start is 'what do you want to students to do and/or learn?' and, following from this: 'how will the students engage with the material?', which leads to the design and development of learning tasks and activities. It is crucial that at this point the assessment tasks clearly link back to the learning outcomes and are appropriate for the learning activities. The assessment can be used for formative and summative purposes. Assessment can be represented as follows:

The assessment process



Once it is clear what needs to be assessed and why, then the form the assessment will take needs to be determined. There are many types of assessment tasks that can be implemented; the factors that will determine choices include:

- the students—how many are there, what is expected of them, how long will the assessment task take?
- the learning outcomes of the subject and how they might be best achieved.

During the year you must set assessment tasks, which ensure that all the learning outcomes of the subject have been assessed internally. Each task you set must include assessment criteria that provide clear guidelines to students as to how, and to what extent, the achievement of the learning outcomes may be demonstrated.

Marking guides and assessment criteria help you with the marking process and ensure that your assessment is consistent across classes. It is important that marking guides and assessment criteria are collectively developed.

Students must complete the assessment tasks set. Each task must provide clear guidelines to students for how the task will be completed and how the criteria will be applied. When you set a task make sure that:

- the requirements of the task are made as clear as possible to the student
- the assessment criteria and performance standards or marking guides are given to the student so that they know what it is that they have to do
- any sources or stimulus material used are clear and appropriate to the task
- instructions are clear and concise
- the language level is appropriate for the grade
- it does not contain gender, cultural or any other bias
- materials and equipment needed are available to students
- adequate time is allowed for completion of the task.

Assessment methods

Although assessment methods and weightings are stipulated in the syllabus, you decide which assessment method to use when assessing the learning outcomes. You should use a variety of assessment methods to suit the purpose of the assessment. Assessment can be classified into four categories:

- tests
- product or project assessments
- performance assessments
- process skills assessments

Because each has limitations, maintaining a balance of assessment methods is very important.

Tests

A ‘test’ is a formal and structured assessment of student achievement and progress, which the teacher administers to the class. Tests are an important aspect of the learning and teaching process if they are integrated into the regular class routine and not treated merely as a summative strategy. They allow students to monitor their progress and provide valuable information for you in planning further learning and teaching activities.

Tests will assist student learning if they are clearly linked to the outcomes. Evidence has shown that several short tests are more effective for student progress than one long test. It is extremely important that tests are marked and that students are given feedback on their performance.

There are many different types of tests. Tests should be designed to find out what students know and about the development of thinking processes and skills. Open questions provide more detailed information about achievement than a question to which there is only one answer.

Principles of designing classroom tests

Tests allow a wide variety of ways for students to demonstrate what they know and can do. Therefore:

- students need to understand the purpose and value of the test
- the test must assess intended outcomes
- clear directions must be given for each section of the test
- the questions should vary from simple to complex
- marks should be awarded for each section
- the question types (true or false, fill-in-the-blank, multiple-choice, extended response, short answer, matching) should be varied.

Tests should:

- be easy to read (and have space between questions to facilitate reading and writing)
- reflect an appropriate reading level
- involve a variety of tasks
- make allowance for students with special needs
- give students some choice in the questions they select
- vary the levels of questions to include gathering, processing and applying information
- provide sufficient time for all students to finish.

Product or project assessments

A ‘project’ can be an assessment task given to an individual student or a group of students on a topic related to the subject. The project results in a ‘product’ that is assessed. The project may involve both in-class and out-of-class research and development. The project should be primarily a learning experience, not solely an assessment task. Because a great deal of time and effort goes into producing a quality product from a project assignment task, you should allow class time to work on the project. A product or project:

- allows the students to formulate their own questions and then try to find answers to them
- provides students with opportunities to use their multiple intelligences to create a product
- allows teachers to assign projects at different levels of difficulty to account for individual learning styles and ability levels
- can be motivating to students
- provides an opportunity for positive interaction and collaboration among peers

- provides an alternative for students who have problems reading and writing
- increases the self-esteem of students who would not get recognition on tests or traditional writing assignments
- allows for students to share their learning and accomplishments with other students, classes, parents, or community members
- can achieve essential learning outcomes through application and transfer.

Assignments

'Assignments' are unsupervised pieces of work that often combine formative and summative assessment tasks. They form a major component of continuous assessment in which more than one assessment item is completed within the term. Any of the methods of assessment can be set as assignments, although restrictions in format, such as word limits and due dates, are often put on the assessment task to increase their practicality.

Investigations

An 'investigation' involves students in a study of an issue or a problem. Teachers may guide students through their study of the issue; or individual students, or groups of students, may choose and develop an issue in negotiation with the teacher. The emphasis in this assessment component is on the student's investigation of the issue in its context by collecting, analysing and commenting on secondary data and information. Students should be encouraged to consider and explore a variety of perspectives as they develop and state their position on the issue. Students may present the investigation for assessment in various forms, including one or a combination of: a written report, an oral presentation, a website, linked documents, multimedia, a video or audio recording.

Criteria for judging performance in the investigation

Students' performance will be judged by the extent to which the student:

- identifies and describes the issue or problem
- describes and explain the causes and effects
- critically analyses information and outlines possible steps leading to a solution or recommendation.

Portfolios

'Portfolios' provide evidence for judgements of student achievement in a range of contexts. A portfolio contains a specific collection of student work or evidence. This collection of work should provide a fair, valid and informative picture of the student's accomplishments.

Computer-based tasks

Using computers to administer student assessment can provide flexibility in the time, location or even the questions being answered of students. The most common type of computer-based assessment is based on multiple-choice questions, which can assist teachers to manage large volumes of marking and feedback.

Process skills assessments

This method of assessment component involves assessing students' understanding of concepts based on the practical skills that can be used, the evaluation of work done and/or the reporting of information. These skills include, for example:

- interpretation skills
- evaluation skills
- reflection skills
- communication skills (such as writing, speaking, and listening).

Types of assessment tasks

Using different assessment tasks is the way to make sure that students are able to demonstrate the range of their abilities in different contexts. Each category has advantages in assessing different learning outcomes. For example, a selected response assessment task, such as a series of multiple-choice questions, is able to assess all areas of mastery of knowledge but only some kinds of reasoning.

Assessment ideas for individual students or groups

Tests	Products or projects	Performances	Process skills
Essay Multiple-choice Matching Short answer	Graphs, charts, diagrams Inventions Models Posters Projects Proposals Research papers Results of surveys Displays Games Maps Questionnaires Simulation game Video tapes	Activities Cooperative learning, group activities Debates Demonstrations Discussions Experiments Explanations Field trips Presentations Reports Role-plays Surveys Warnings	Analysing Observing Evaluating Predicting Interpreting Hypothesising Investigating Explaining Classifying Experimenting Estimating Communicating Researching Designing Manipulating Collecting data Synthesising Critiquing Concept mapping Interviewing

Feedback

When you assess the task, remember that feedback will help the student understand why he or she received the result and how to do better next time. Feedback should be:

- *constructive*, so that students feel encouraged and motivated to improve

- *timely*, so that students can use it for subsequent learning
- *prompt*, so that students can remember what they did and thought at the time
- *focused on achievement*, not effort. Assess the work, not the student
- *specific to the unit learning outcomes*, so that assessment is clearly linked to learning.

Types of feedback

Feedback can be:

- *informal or indirect*: such as verbal feedback in the classroom to the whole class, or person to person
- *formal or direct*: in writing, such as checklists or written commentary to individual student either in written or verbal form
- *formative*: given during the topic with the purpose of helping the student know how to improve
- *summative*: given at the end of the topic with the purpose of letting the students know what they have achieved.

Who assesses?

Teacher assessment

Assessment is a continuous process. You should:

always ask questions that are relevant to the outcomes and content

use frequent formative tests or quizzes

check understanding of the previous lesson at the beginning of the next lesson, through questions or a short quiz

constantly mark or check the students' written exercises, class tests, homework activities and so on

use appropriate assessment methods to assess the tasks.

Frequency of assessment

You should schedule specific assessment tasks to fit in with teaching of the content of each unit that is being assessed. Some assessment tasks might be programmed to be undertaken early in the unit, others at the end of the unit. You should take care not to overload classes with assessment tasks at the end of the term.

Judging student performance

Student achievement is recorded and reported against standards. You must use performance standards or marking guides—examples of which are provided in this teacher guide—when making a decision about the achievement of your students in relation to the learning outcomes. The performance standards describe the level at which the student has to be working to achieve a particular standard or mark. Students should always have access to a copy of the assessment criteria and the performance standards so that they know what they have to know and be able to do to get a good mark in a particular task. The performance standards will help you in your marking and will help your students improve their performance in the

future. They are useful when providing feedback to students as they explain what the student needs to do to improve.

Moderation

To make sure that you are interpreting the performance standards correctly when assessing your students, it is important to undertake Advanced Mathematics moderation of student work within your school and with teachers of nearby schools. To moderate student work, a common assessment task must be used and a marking scheme developed so that all students complete the same task under the same conditions, and all teachers use the same marking scheme. Teachers can then compare (moderate) the students' work and come to a common understanding of the performance standards and the requirements for a particular mark or level of achievement. Moderation enables you to be sure that your understanding of the required standards for levels of achievement is similar to the understanding of other teachers and that you are assessing students at the appropriate level.

Self-assessment and peer assessment

Self-and peer assessment helps students to understand more about how to learn. Students should be provided with opportunities to assess their own learning (self-assessment) and the learning of others (peer assessment) according to set criteria. Self-assessment and peer assessment:

- continue the learning cycle by making assessment part of learning
- show students their strengths and areas where they need to improve
- engage students actively in the assessment process
- enable students to be responsible for the learning
- help to build self-esteem though a realistic view of their abilities
- help students understand the assessment criteria and performance standards.

Managing assessment tasks for Advanced Mathematics

Usually, the teacher marks assessment tasks. To reduce work you need to develop a strategic approach to assessment and develop efficiencies in marking. In Advanced Mathematics there are a number of assessment tasks that may be new to teachers and students. Suggestions on how to manage some of these tasks to minimise marking or presentation time include:

Develop efficiency in marking

Clarify assessment criteria

Plan the assessment task carefully, and ensure that all students are informed of the criteria before they begin. Discuss the task and its criteria in class, giving examples of what is required. Distribute a written copy of the instructions and the criteria, or put them on the board. Making the assessment criteria explicit speeds marking and simplifies feedback.

Supply guidelines on what is required for the task

This reduces the amount of time that may be wasted evaluating student work that is irrelevant.

Use attachment sheets such as marking guides

An assignment attachment sheet, which is returned with the assessed work, rates aspects of the task with a brief comment. Such a system enables each student's work to be marked systematically and quickly. This strategy can be applied to posters, presentations and performances.

Assess in class

Use class time to carry out and to assess tasks. Performances or art works, marked by you or the students, enable instant developmental evaluation and feedback. Brief assessments of projects, stages of the design process, or practical work take less time to mark and are useful because they give immediate feedback to students on their progress and allow you to mark the project in stages with minimum effort.

Give feedback to the whole class

Feedback to the whole class can cut down on the amount of individual feedback required. On returning assessed work, emphasise the criteria for judging the work, discuss the characteristics of good and bad answers, and highlight common strengths and weaknesses.

Set group work alternatives

Assess one performance per group. The student's mark is the group mark, but may include a component based on the contribution of the individual. A strategy for allocating an individual mark includes each member of the group using criteria to evaluate the relative contributions of individuals, with the marks averaged for the individual.

Set clear deadlines

Set aside a time for marking. Be careful about extending this period (by allowing students to hand in work late).

Shift the responsibility

Introduce self-assessment and peer assessment

Develop in students the skills to evaluate their own work and that of their peers. With the students, use the assessment criteria against which work is judged, highlighting strengths and weaknesses. Self-assessment increases the feedback to students. It can supplement or replace teacher assessment.

Treat each task differently

Every piece of work need not be evaluated to the same degree; a mark need not be the outcome in every case; and not every piece of student work needs to contribute to the final grade. Assessment is designed to enhance the learning and teaching experience for the teacher and the learner, not just to give marks.

Sample assessment tasks

All assessment tasks must test whether or not the student has achieved the outcome or outcomes. Each task must have clear and detailed instructions. Students must know exactly what they have to do. You should develop marking guides when you are marking tasks to ensure consistency of your assessment.

Grade 11

The following is an example of an assessment task and marking guide for unit 11.3. The sample task and assessment criteria can be used to assess the outcomes of this unit. Teachers can use the sample to develop other assessment tasks, criteria and performance standards.

Sample task: Statistical survey

Students are to conduct a survey on an area of interest. They will collect, process, tabulate and analyse data. This task can be done in groups of four to six students.

Learning outcomes

Students can:

1. communicate mathematical processes and results verbally and in writing
6. apply knowledge of statistics and probability to communicate, justify, predict and critically analyse findings and draw conclusions
9. plan, organise and carry out mathematical activities individually and cooperatively.

Assessment criteria

Students will be assessed on the extent to which they can:

- demonstrate appropriate investigation skills
- choose and apply relevant mathematical techniques
- make an effective communication of the survey results.

Task specifications

- decide on an area of interest on which they would like to conduct a survey
- choose methods for gathering information
- conduct the survey
- process data collected
- present analysis of data and recommendations

Performance standards for statistical survey				
Assessment criteria	**Very high achievement**	**High achievement**	**Satisfactory achievement**	**Low achievement**
Demonstrate appropriate investigation skills	• Consistently stays focused on the task • Very self-directed • Actively collects information and creates insightful solutions to problems • Uses a wide range of resources	• Focused on the task most of the time • Collects information and finds standard solutions to problems • Uses at least two different resources	• Focused on the task some of the time • Collects information and finds solutions to problems with some assistance • Uses at least two different resources	• Rarely focuses on the task • Collects some information without providing adequate solutions • Uses at most one resource
Choose and apply relevant mathematical techniques	• Appropriate and efficient mathematical techniques used at all times • Solution contains no mathematical errors, or almost none	• Uses appropriate and effective mathematical techniques • Solution contains few mathematical errors	• Sometimes uses appropriate and effective mathematical techniques but does not do it consistently • Solution contains some major mathematical errors	• Rarely uses an appropriate mathematical techniques • Solution contains many mathematical errors
Make an effective communication of the survey results	• Work is presented in a well organised fashion that is easy to read or listen to and is easy to understand • All project results communicated clearly	• Work is presented in an organised fashion and is mostly easy to read and understand • Most project results communicated	• Work is presented in a reasonably organised fashion but is not always easy to read or understand • Some project results communicated	• Work appears sloppy and unorganised • It is hard to know what information goes together • Few if any project results communicated

Sample marking guide

Marking guides, like the one below, should be used to assess the tasks you set. You can tick the appropriate box, look at the performance standards and the students' overall achievement and give an on-balance assessment.

If, for example, the students gets two ticks in the 'Very high achievement' (VHA) column, most of their ticks in the 'High achievement' (HA) column, several ticks in the 'Satisfactory achievement' column and one tick in the 'Low achievement' column, then on balance you would give the students a 'High achievement' and a mark between 70 and 89.

Sample marking guide for an investigation or project

11.3 Statistical survey					
Criteria		**VHA**	**HA**	**SA**	**LA**
Demonstrate appropriate investigation skills	• Suitability of topic chosen • Planning process • Respond to advice				
Choose and apply relevant mathematical techniques	• Method chosen for data collection • Conducting the survey • Tabulation of results • Analysis of findings • Recommendations				
Make an effective communication of the survey results	• Suitability for purpose and audience • Clarity • Use of visual and other aids				

Sample self evaluation sheet

Work-in progress: Self-evaluation

- What were the key things you learned or gained from this? (information, skills, ideas, experience)
- How effective was your group during planning, developing and rehearsing your presentation (give reasons)
- How effective were you during the planning, developing and rehearsal phases of your work? (give reasons).
- What were your group's strengths and weaknesses in
  - collaboration
  - communication
  - research?
- What are your own strengths and weaknesses in
  - collaboration
  - communication
  - research?
- What does your group need to develop next in order to improve the quality of your presentation?
- What will you do to continue to these goals?

Grade 12

The following is an example of an assessment task and marking guide for any Grade 12 unit. The sample task and assessment criteria can be used to assess the outcomes of the chosen unit. Teachers can use the sample to develop other assessment tasks, criteria and performance standards.

Sample task: Modelling problem

Use mathematical models to solve real-world problems.

Learning outcomes

Students can:

1. communicate mathematical processes and results verbally and in writing
7. describe and explain interrelationships between mathematical concepts
8. analyse mathematical situations and construct and apply logical arguments in resolution of mathematical problems.

Assessment criteria

Students will be assessed on the extent to which they can:

- demonstrate appropriate investigation skills
- choose and apply relevant mathematical techniques
- make an effective communication of the survey results.

Task specifications

The steps in the modelling process can be summarised as follows:

1. Specify the real world problem:
   - be clear and concise
   - what assumptions need to be made?
2. Formulate the mathematical problem:
   - use symbols
   - set up equations
3. Solve the mathematical problem:
   - solve equations
4. Make sense of the solution:
   - do all solutions make sense?
   - what does the solution mean in real terms?
5. Test the model:
   - put the solution into practice if practical
   - check the solution to see if it works.
6. Communicate the solution:
   - write a report; present it to class

Performance standards for modelling problem				
Assessment criteria	**Very high achievement**	**High achievement**	**Satisfactory achievement**	**Low achievement**
Demonstrate appropriate investigation skills	• Consistently stays focused on the task • Very self-directed • Actively collects information and creates insightful solutions to problems • Uses a wide range of resources	• Focused on the task most of the time • Collects information and finds standard solutions to problems • Uses at least two different resources	• Focused on the task some of the time • Collects information and finds solutions to problems with some assistance • Uses at least two different resources	• Rarely focuses on the task • Collects some information without providing adequate solutions • Uses at most one resource
Choose and apply relevant mathematical techniques	• Appropriate and efficient mathematical techniques used at all times • Solution contains no mathematical errors, or almost none	• Usually uses appropriate and effective mathematical techniques • Solution contains few mathematical errors	• Sometimes uses appropriate and effective mathematical techniques but does not do it consistently • Solution contains some major mathematical errors	• Rarely uses an appropriate mathematical techniques • Solution contains many mathematical errors
Make an effective communication of the survey results	• Work is presented in a well organised fashion that is easy to read or listen to and is easy to understand • All project results communicated clearly	• Work is presented in an organised fashion and is mostly easy to read and understand • Most project results communicated	• Work is presented in a reasonably organised fashion but is not always easy to read or understand • Some project results communicated	• Work appears sloppy and unorganised • It is hard to know what information goes together • Few if any project results communicated

Sample marking guide

Marking guides, like the one below, should be used to assess the tasks you set. You can tick the appropriate box, look at the performance standards and the students' overall achievement and give an on-balance assessment. If, for example, the students gets two ticks in the 'Very high achievement' (VHA) column, most of their ticks in the 'High achievement' (HA) column, several ticks in the 'Satisfactory achievement' column and one tick in the 'Low achievement' column, then on balance you would give the students a 'High achievement' and a mark between 70 and 89.

Sample marking guide for modelling problem

Modelling problem					
Criteria		**VHA**	**HA**	**SA**	**LA**
Demonstrate appropriate investigation skills	• Is focused • Can work on his or her own • Collects information • Uses different resources				
Choose and apply relevant mathematical techniques	• Use symbols • Set up equations • Solve equations • Do all solutions make sense? • What does the solution mean in real terms? • Check the solution to see if it works				
Make an effective communication of the survey results	• Organisation • Easy to read and understand • Clear and concise • present it to class				

Learning activities and assessment tasks

Examples of learning activities and assessment tasks for each of the Advanced Mathematics units are provided in the following sections. Some examples are explained in detail.

Grade 11

11.1 Number and Application

Suggested activities

Integers

Exploratory exercise (calculators may be used for this exercise)

1. Pick any number from the list
   2, 3, 4, 5, 6
   And multiply it by 123456789 and then by 9.Repeat using another number. Without calculating first, what would be the number if you used the number 8?
2. A prime number is any number greater than 1 which has only two factors, itself and 1.
   So 23 is a prime number because 23 = 23 x 1 and there are no other factors. All other numbers (except 1) are called composite numbers.
   - List the numbers between (inclusive):
     (a) $2^2 and 3^2$ (b) $3^2 and 4^2$
     (c) $5^2 and 6^2$ (d) $7^2 and 8^2$
   - Circle the prime and cross out the composite numbers
   - Guess how many prime numbers there are between 8^2 and 9^2. Check your guess to see how close it was.
3. Find:
   (a) 1 +3 (b) 1+3+ 5
   (c) 1 + 3 +5 7 (d) 1 + 3 + 5 + 7+9
   - What is the name of the number obtained?
     Use this to find: 1+ 3 + 5 + 7 + … + 31
4. Write down all the factors of 6 apart from 6 itself and find their sum.
   - What do you notice? This is called a perfect number.
   - Find the next perfect number—it has two digits.
5. Write down all the factors of 8 apart from 8 itself and find their sum.
   - What do you notice? Is the sum greater than 8 or less than 8? If the sum is greater, than 8 is said to be an abundant number; if it is less, 8 is called a deficient number.
   - Is 8 abundant or deficient? Find the first number after 6 which is abundant and the first which is deficient.
6. Write down all the factors of 220 apart from 220 itself and find their sum. Now write down all the factors of 284 from 284 itself and find their sum.

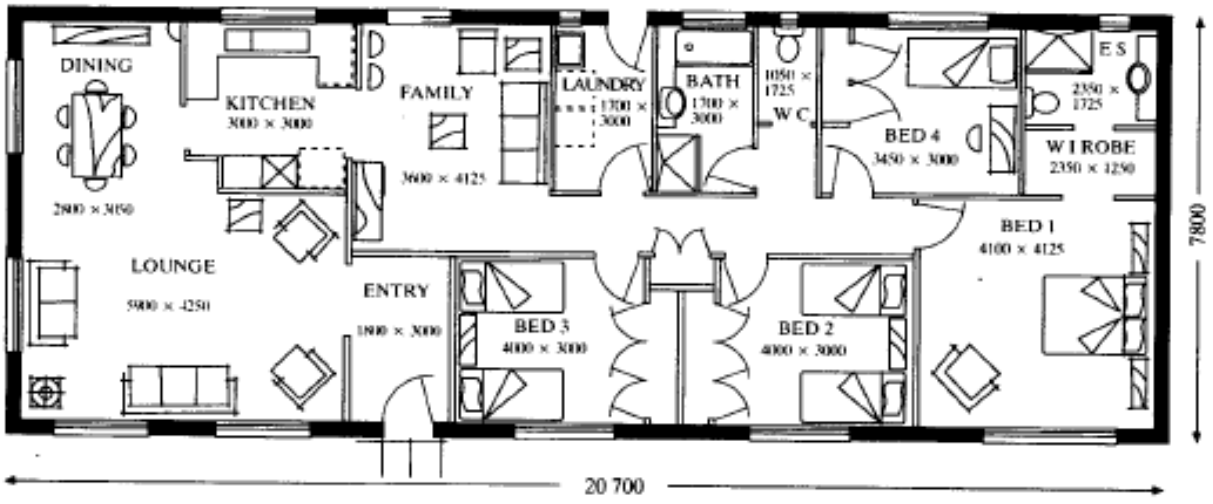
Such numbers are called friendly (or amicable) numbers. Are 18 and 21 friendly?

Suggested assessment tasks

Ratio and proportion: Scale drawing)

First make a sketch (architectural projection) of your house. Don't worry about too much detail, just include major features such as windows and doors. Now draw a plan of the house by following these steps:

1. Draw a sketch of the perimeter of the house, including all windows and doors (don't worry about doing it to scale yet).
2. Mark in the internal walls and doorways. Measure the internal room dimensions and mark these on the sketch as you go.
3. Work out the overall dimensions of the house and mark these on your sketch. To help you do this, you can assume that internal walls are 100 mm thick and brick veneer walls are 250 mm thick.
4. Draw the plan to scale on graph paper, using the scale 1:100 (this is the scale most often used for house plans).
5. When the plan is complete, draw front, rear and side elevations using the same scale. You can now try your hand at some hypothetical home improvements:
6. Redraw the outside walls of the house from the scale drawing you have already completed.
7. Examine the positions of the internal walls and partitions and determine if any could be move (or removed) to improve the design of the house. You will also probably have to move some windows and doors.
8. Write a short explanation of how your changes improve the design of the house (see below).



11.2 Graphs and Functions

Suggested activities

- use logarithms to solve indicial equations and inequalities
- exponential growth and decay

Logarithm

This activity is suitably done on a flat hard surface such as a cemented basketball court, using a round bouncing object like a tennis ball. A metre ruler or accurately marked cardboard or something similar could be used to measure height of the bouncing ball.

Steps

1. drop the ball from a selected height
2. measure the height of the rebounding ball
3. find % (fraction) of the rebound on original height
4. using the formula $h = A \times a^n$ of the height 'h' of a ball after 'n' bounces where
   - A is the original height
   - a is the % of the height reached by the ball on previous bounce
   - n is the number of bounces
5. write the formula for 'h' in terms of 'n'
6. determine the height when given a certain number of bounces
7. determine the number of bounces when given a certain height (use log)

Suggested assessment tasks

- spread of disease (logistic model).
- market updates of new products (surge model).

Teeth, plaque and mouthwashes (Exponential growth and decay)

At the end of each period in which mouthwash was used, the dental student examined the teeth of sixteen volunteers, giving each tooth a score between 0 and 4 according to the amount of plaque, a score of four representing the highest level of plaque. An average value was then calculated for each volunteer when using each mouthwash. The average scores were called Score A (referring to old mouthwash A) and Score B (referring to new mouthwash B).

The scores for each volunteer is shown in the following table.

Volunteer	Score A Old mouthwash	Score B New mouthwash
1	3.36	3.16
2	3.55	2.70
3	3.38	3.00
4	3.63	2.96
5	2.86	2.79
6	3.38	3.73
7	3.48	3.39
8	3.66	3.29
9	3.46	3.54
10	3.48	3.71
11	3. 89	3.43
12	3.21	2.64
13	3.38	3.13
14	3.52	3.45
15	3.18	2.70
16	3.25	3.00

Analyse these data in order to reach a conjecture regarding whether or not they suggest that the new mouthwash performs better than the old preparation with respect to the reduction of plaque on teeth.

You must clearly state your conjecture and show all evidence you have found that supports it.

11.3 Managing Data

Elaboration of content

Managing data

- construct an unordered stem plot by listing the leaf digits in the order they appear in the data.

Unordered

Stem	Leaf
2	9,3,9,5,8,8,9
3	2,7,2,9,2,0,3,4,5,5,3,0

rearrange the leaf digits in numerical order to create an ordered stem plot:

Ordered

Stem	Leaf
2	3,5,8,8,9,9,9
3	0,0,2,2,2,3,3,4,5,5,7,8,9

include a key so that the data can be understood; for example, key: 2/3 = 23

Probability

- do practical experiments involving coin tossing, dice rolling, and picking cards from a pack
- identify sample spaces and determine the outcomes
- comment critically on statements involving probability, such as 'since it either rains or is fine, the probability of a fine day is 50–50'.
- investigate expressions used in other disciplines and in everyday life to describe likely or unlikely events; for example, 'once in a blue moon'
- statements involving the language of probability could be collected from various media sources and discussed.
- discuss where probability is used in real life. Provide examples such as:
  - evaluating n ! where n < 10
  - evaluate values of n Pr where n <10 with calculator.
  - relate permutation to ordering
- evaluate values given n Cr where n <100:
  - calculator practice n ≤ 100
- project: students given cards labelled 1 to 10 and cards labelled A to E and they write all possible orders.

Suggested activities

Survey

- choose a problem to investigate where you can first do a survey and then run a trial. It might be a new idea for class fund-raising activity or social function.
  - tabulate the results from both the survey and the trial or experiment and then compare them. In the survey you test a hypothesis using a sample and in the trial or experiment you measure the effect of changing a variable
- In practice, particularly in marketing, a survey is conducted and if the results are reasonably favourable, a trail may then be conducted before money is invested in full scale production. Find examples of different strategies involved in marketing new products
- discuss and define sampling procedure
  - non-random sampling
  - accessibility sampling
  - volunteer sampling
  - judgemental sampling

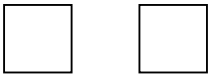
- quota sampling
- random sampling
- stratified sampling

Combinations

You will need five different coloured objects, say 5 blocks.

Steps

1. Take 3 blocks, red, yellow and blue, and arrange them in a row.
   - how many arrangements are there?
   - list the arrangements.
   - jumble them up and select 2.



   - how many ways are there of filling the first box?
   - how many ways are there of filling the second box?
   - list them ordered in groups of 2. If order doesn't matter and you are only interested in the composition of the selection—RY is the same selection as YR—go through list crossing out equivalent selections
   - how many selections of 2 from 3are there?
2. Add another block. Suppose you now have red, yellow, blue and green blocks.
   - how many arrangements of 3 objects chosen from the 4 are there?
   - list these ordered groups of 3 (a tree diagram may help).
   - go through the list and cross out equivalent selections.
   - how many selections of 3 objects chosen from 4 are there?
   - if ${}^4P_3$ gives the number of arrangements of 3 from 4, what must you divide by to get the number of selections of 3 from 4. This is called ${}^4C_3$ or $\binom{3}{4}$.
3. With 5 objects (R, Y, B, G and W),
   - how many arrangements of 3 chosen from5 are there?
   - how many selections of 3 chosen from 5 are there?

Data collection and presentation

Students identify a problem/issue of interest to them that can be investigated by the collection and subsequent analysis of data that they are able to collect or is available to them in either raw or secondary form.

The problem/issue must be able to be categorized as either of the examples shown in the following table:

Response	Explanatory
one interval scale variable	one nominal/ordinal scale variable
one nominal/ordinal variable	one nominal/ordinal variable

After discussion collect the necessary data
Analyse the data in order to construct an argument that results in a conjecture

Format of the presentation
Students may choose how they wish to present their findings. It may be a newspaper or magazine article, a video (e.g. like a news item), a poster, a Power Point presentation or simply a traditional presentation on A-4 paper.

Specifications
The following must be included in the presentation, no matter the medium chosen. They are:

- a description of the problem or issue investigated
- a simple account of the method have employed to carry out the investigation
- the analysis carried out. This includes a copy of data, any graphics and summary statistics produced and the argument formed to support the conjecture
- the conjecture
- a discussion of any weaknesses in the method that may cause the conjecture to be suspect.

Suggested assessment tasks

Example

- Sample space (S) of an element is defined by S = factors of 36
  - list the elements of (S)
  - find n(S)
- Two male learners M1 and M2 and two female learners F1 and F2 are called to the teachers table
  - draw a tree diagram that illustrates the possible outcome when the four learners line up at the table.
  - write down n(S).
  - find P (M1 is second in line)
  - find P (F1 is first and F2 is third in line)
  - find P (F1 and F2 are standing) between M1 and M2)

Data collection and presentation: Survey

Imagine the following situation:

- Some students in your class want ice cream to be available in the school canteen and in discussion, claim that some students would buy ice cream if it was available. The canteen supervisor requires a better basis for ordering the ice cream so the class decides to conduct a survey.

Suggested questions for the survey form

1. Do you like ice cream?
2. Would you buy ice cream if it was available at the school canteen?
3. Would you buy ice cream at least twice a week if it was available at the canteen?

Suggested methods of surveying

- Ask the first 100 students who go out of the school gate in the afternoon.
- Assign a number to all the students in the school and then pick numbers out of hat or box to choose 100.
- Find the proportion at each level and choose the sample from each year levelling this proportion as they go out of the school gate.
- Assign numbers in each year level and choose the proportion calculated in method 3 randomly.
- Choose classrooms randomly and ask every second or third student in these rooms until 100 have been asked.
- Discuss the advantages and disadvantages of each of the suggested sampling procedures.
- Tabulate the results and draw conclusions.

11.4 Geometry

Suggested activities

Distance between two points

- Given the fact that the length of an arc on a great circle subtending an angle of 1° at the centre of the earth is equal to approximately 111.6 km; the Tropic of Capricorn is approximately 23°S of the equator; and the Tropic of Cancer approximately 23°N of the equator:
  - find distance between the two tropics along the meridian of longitude
  - calculate distance between the North and the South poles
  - calculate circumference of the earth at the equator

Steps

1. find a globe or a world map
2. identify the two poles, two tropics and the equator
3. identify the meridian of longitude
4. find the distance between the two tropics along the meridian of longitude
5. calculate the distance between the North and the South poles
6. calculate the circumference of the earth at the equator

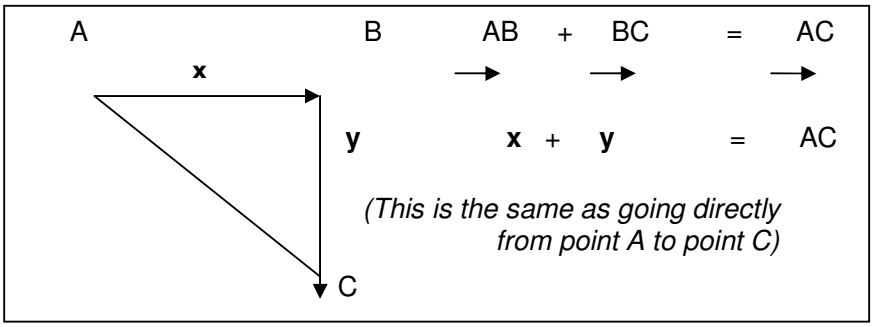
Representing vectors

- Get students out of the classroom and give them directions to which they must turn and walk certain steps. To start, they face the sunrise (east).

Steps

1. face the east (sunrise) and take x steps towards the east
2. turn 90° to the right (right turn)
3. take y steps towards the south
4. find the vector AC

You can explain that vector quantity involves magnitude (length, distance and size) as well as the direction of the movement of the object.



Suggested assessment tasks

- Draw a scale model of an object such as plan of a house.
- Construct concentric circles and compare magnitude of similar cyclic quadrilaterals sides and angles.
- Design group work on Mercator Projection.

12.1 Patterns and Algebra

Suggested activities

Sequences and series

Steps

1. Take a large sheet of paper; fold it in half and in half again. Cut the folded corner. Open up the paper and count the number of 'holes'. Refold the paper and then fold once more. Cut the folded corner. Open up the paper and count the number of 'holes'. Repeat this process until the paper is too small to fold and cut any more.
2. Copy and complete the following table:

Number of folds	Number of holes
2	
3	
4	
5	
6	

3. From the pattern found in (2), how many holes would be found by 20 folds? (*Hint: investigate the pattern 1, 2, 4,…*)

Suggested assessment tasks

Motion graph

This activity could be done in groups of 3 to 4 where a marble or any other smooth, round object could be rolled down a ramp (which can be built using two strips of thin timber nailed to plywood). The overall length of the ramp can be determined by the teacher.

Steps

1. draw a table to record distance and time for a marble to roll down the ramp
2. students roll a marble down the ramp and record the distance-time relation into the table
3. plot this information on a graph
4. use this graph to calculate further distances or times

This activity can be extended to plotting the graph of rolling the marble up and down the ramp. Using these activities, teachers can assess students' ability to:

- sketch graphs and investigate the graph of a function in plotting points
- use graphical methods to investigate a pattern in data and identify its algebraic form where appropriate.

12.2 Trigonometry and Vectors

Suggested activities

- explain the key features used in drawing graphs of trigonometric functions; for example:

  $y = \sin x$, $y = \sin x$, $y = a \sin px$,

  $y = a \sin(px + n\pi 0 + h$

  – what do the variables represent?
- draw graphs of basic and further trigonometric functions and identify the amplitude, period, phase shift and transformation of graphs

- demonstrate how to find the speed of a river by floating a log in a creek, or a constructed stream of running water:
  - measure how far it has gone down the stream of water
  - then use v = d/t to calculate the speed of the water
- teacher provides plane speed, wind speed, distance between two Papua New Guinean towns and altitude from DCA office nearby to students for actual scale drawing and calculation of the velocity and direction of flight with respect to wind
  - discuss importance of weather before flight or sailing
- in an investigation of weather patterns, temperature, *T°C*, in Kiunga Secondary was found to fluctuate approximately according to the rule:

  $T = 25 + 6 \sin 0.1\Pi t$ where t is the number of hours after 10.00 am
  - sketch a graph of the temperature fluctuation for a sufficient number of hours to be able to determine the maximum and minimum temperatures for that day
  - use the graph to determine the maximum and minimum temperatures
  - when did they occur?
  - at what time was the temperature (i) $27^0 C$ (ii) $20^o C$?

Suggested assessment tasks

- Use newspaper or other source to find the time and depth of each high and low tide in a coastal province over a period of 3 days:
  - plot the points generated, showing time on the horizontal axis and depth on the vertical axis. Join the points with a smooth curve
  - over what period of time does there appear to be a repetition of graph?
- Measure the velocity of a flowing river and/or sea by finding the distance of the river or distance between 2 points in the sea and the time taken to swim across the river or sea. Draw a vector diagram of the actual velocity of swimmer with river or sea current speed; or
- Draw and calculate actual plane speed from the given data.

Unit 12.3 Calculus

Suggested activities

- discuss gradients
- discuss all differentiation rules
  - basic exponents
  - product
  - quotient
  - chain
- recap definition of tangent and normal
- recap gradient formula for normal and parallel lines

$$m_1 = m_2 , m_1 = -\frac{1}{m_2}$$

- discuss maximum, minimum and points of inflection
- give table of derivative for basic trigonometric function

Function	Derivative
$\sin x$	$\cos x$
$\cos x$	$\sin x$
$\tan x$	$\sec^2 x$
$csc\, x$	$-\csc x \cot x$
$\sec x$	$\sec x \tan x$
$\cot x$	$-\csc^2 x$

Rates of change, differentiation and anti-differentiation

Analysis question

Consider the situation where an 18 m high spire is to be built with a vertical cross section in the form of the graphs of two cubic polynomial functions, as illustrated below:

$f(x) = 0.25(x + 4)^3 + 2$, $x \in [-6, 0]$ and $g(x) = -0.25(x - 4)^3 + 2$, $x \in (0, 6]$

- If the graph is not given, students could be asked to draw the graph of the hybrid function. In the first part of the analysis question, students could be asked to find the average rate of change in height between several pairs of points, for example, between $x = 0$ and $x = 6$.
- In the second part of the analysis question, they could then be asked to determine the gradient, correct to a specified accuracy of the sides of the spire at several points, for example when $x = 0$, 2 and 6.
- In the third part of the analysis question, students could be asked to find an approximation to the cross-sectional area using a series of rectangles, for example left and right rectangles of constant base lengths of 0.5 m.
- In the final part of the analysis question, students could be asked to find the anti-derivative function, F, of f, and compare computed values of this to the area approximation obtained earlier, for example:

  $F(0) - F(-6)$ to the rectangle approximations from $x = -6$ to $x = 0$

Suggested assessment tasks

Rates of change in calculus

- Students are asked to determine the time it takes to fill a variety of shaped containers with water from a tap that pours at a constant rate of 0.1L/s, graph the corresponding depth of water versus time graph and present their findings in a brief written report. For example, the containers could be:
  - *rectangular* prism 140 cm long, 60 cm wide and 40 cm high

  - *cylinder* 40 cm deep and has a radius of 40 cm
  - *trapezoidal prism* 140 cm long, 50 cm wide at the base, 70 cm wide at the top and 40 cm high
  - *inverted truncated rectangular prism* (a simple common bath shape) that is 40 cm deep, 150 cm long at the top and 130 cm long at the base, 50 cm wide at the base and 70cm wide at the top
  - the *dimensions of a bath* and determining a reasonable flow rate from your tap (use a measuring jug and determine the volume over 10 seconds).
- Assignment on anti-derivative, indefinite and define integrals.
- Project: Students given a specified weighted object. They process required calculations involved in 'Motion of a Projectile' and also area covered.

Total: 50 marks

Recording and reporting

All schools must meet the requirements for maintaining and submitting student records as specified in the *Grade 12 Assessment, Examination and Certification Handbook*.

Recording and reporting student achievement

When recording and reporting student achievement you must record the achievement of the students in each unit and then, at the end of the year, make a final judgement about the overall achievement, or progress towards achievement, of the learning outcomes. To help you do this, descriptions of the levels of achievement of the learning outcomes are provided in the 'Learning outcome performance standards' table.

When reporting to parents, the school will determine the method of recording and reporting. In an outcomes-based system, student results should be reported as levels of achievement rather than marks.

Remember that the final school-based mark will be statistically moderated using the external exam results. The students' overall level of achievement may change.

Levels of achievement

The level of achievement of the learning outcomes is determined by the students' performance in the assessment tasks. Marks are given for each assessment task with a total of 100 marks for each 10-week unit, or 50 marks for each five-week unit. The marks show the student's level of achievement in the unit, and hence their progress towards achievement of the learning outcomes. There are five levels of achievement:

- Very high achievement
- High achievement
- Satisfactory achievement
- Low achievement
- Below minimum standard.

A very high achievement means, overall, that the student has an extensive knowledge and understanding of the content and can readily apply this knowledge.

In addition, the student has achieved a very high level of competence in the processes and skills and can apply these skills to new situations.

A high achievement means, overall, that the student has a thorough knowledge and understanding of the content and a high level of competence in the processes and skills.

In addition, the student is able to apply this knowledge and these skills to most situations.

A satisfactory achievement means, overall, that the student has a sound knowledge and understanding of the main areas of content and has achieved an adequate level of competence in the processes and skills.

A low achievement means, overall, that the student has a basic knowledge and some understanding of the content and has achieved a limited or very limited level of competence in the processes and skills.

Below the minimum standard means that the student has provided insufficient evidence to demonstrate achievement of the learning outcomes.

Achievement level					
Total marks	**Very high achievement**	**High achievement**	**Satisfactory achievement**	**Low achievement**	**Below minimum standard**
600	540–600	420–539	300–419	120–299	0–119
500	450–500	350–449	250–349	100–249	0–99
400	360–400	280–359	200–279	80–199	0–79
300	270–300	210–269	150–209	60–149	0–59
200	180–200	140–179	100–139	40–99	0–39
100	90–100	70–89	50–69	20–49	0–19
60	54–60	42–53	30–41	12–29	0–11
50	45–50	35–44	25–34	10–24	0–9
40	36–40	28–35	20–27	8–19	0–7

Sample format for recording Advanced Mathematics assessment task results over two years

Student name:

Grade 11 assessment task results			
Unit	**Assessment task**	**Mark**	**Student mark**
11.1	Measurement assignment		
11.2	Graphs group project	20%	
	Test	20%	
11.3	Survey	15%	
	Test	15%	
11.4	Circle investigation	15%	
	Test	15%	
	Total marks Grade 11	**300**	

Student name:

Grade 12 assessment task results			
Unit	**Assessment task**	**Marks**	**Student mark**
12.1		15%	
12.2		15%	
		20%	
	Total marks Grade 11	**300**	
	Total marks Grade 11 and 12	**600**	

Learning outcomes and levels of achievement

Levels of achievement in Grade 11 and Grade 12 are recorded and reported against the learning outcomes. The performance standards for the levels of achievement are described in the table on pages 13 and 14.

Steps for awarding final student level of achievement

1. Assess tasks using performance standards and assessment criteria and marking guides.
2. Record results for each task.
3. Add marks to achieve a unit result and term result.
4. Add term marks to get a year result.
5. Determine the overall achievement using the achievement level grid.
6. Report results using the learning outcome performance standards.

The following is an example of reporting using the learning outcomes performance standards descriptors.

Using the learning outcomes performance standards descriptors

Student: Subject: School-based assessment:	**Lena Kili** **Advanced Mathematics** **High achievement**

This assessment means that Lena:
Communicates complex mathematical processes and results using an extensive range of written, graphic and oral forms
Independently measures and use a range of appropriate techniques to estimate and calculate physical quantities
Independently applies knowledge of numbers and their relationships to investigate a wide range of contexts
Independently identifies, interprets, describes and represents various functional relationships to solve a range of problems in real and simulated contexts
Gives detailed descriptions and properties of 2-dimensional shapes and 3-dimensional objects in a variety of orientations and positions
Critically analyses, evaluates and interprets statistical data and information
Demonstrates extensive knowledge and understanding of a wide range of interrelationships between mathematical concepts
Gives logical and detailed explanations to analyse a range of mathematical situations and apply logical arguments in resolution of mathematical problems
Demonstrates excellent planning and organising skills to carry out a wide range of mathematical activities

Resources

Advanced Mathematics becomes more interesting and meaningful when you use a variety of resources and local materials in your teaching.

You should always try to adapt, improvise, make, find or write material that will be useful for lessons. Advanced Mathematics can be taught without expensive equipment by making use of what is around you, though there are some equipment and materials that are essential to teach the Advanced Mathematics syllabus.

Types of Advanced Mathematics resources

Materials and equipment

- calculators
- slide projector
- audiovisual aids
- textbooks, reference books
- magazines
- diagrams, posters
- worksheets, information sheets
- pamphlets, brochures
- television and radio broadcasts
- video, film, film strips
- computer software
- pictures, photographs
- newspapers

General guidelines for selecting and using resources

How effective a resource is depends on whether it is suitable for the knowledge or skill to be learned and the attitude of the students. Classroom organisation is the key to using resources successfully. You need to:

- prepare thoroughly. Make sure that you are familiar with the resource so that you use it with confidence and assurance. If equipment is involved, check that it is in working order, make sure that you know how to operate it and that it is available when you need it.
- use the resource at the right place and time—it should fit in with the flow and sequence of the lesson and serve a definite teaching purpose.
- (if the resource is radio, film, video or television), introduce the program by outlining the content. You might also set some questions to guide listening or viewing. Follow up after using the resource, by discussing and drawing appropriate conclusions.

References

Note: These reference books and websites are also useful teacher resources.

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Simpson, N and Rowland, R 2003, *Maths Quest Year 12 Maths B for Queensland (Advanced),* Jacaranda Wiley, Milton Qld.

SSABSA 2002, *Stage 2 Mathematical Methods. Curriculum Statement 2007*, SSABSA, Adelaide.

SSABSA 2005, *Mathematics Learning Area Manual 2005*, SSABSA, Adelaide.

Useful websites

http://education.pwv.gov.za

http://math247.jot.com/WikiHome/Library

http://www .aaamath.com/

http://www.aamt.edu.au/2007

http:// www.bbc.co.uk/schools/

http://www.blackdouglas.com.au/taskcentre/

http://www.boardofstudies.nsw.edu.au

http://www.counton.org/

http://www.firstinmath.com

http://www.flinders.edu.au/science21/

http://www.nrich.maths.org

http://www.shodor.org/

http://www.srl.rmit.edu.au.mav

www.abacus.com.au

www.lat-olm.com.au

www.mathsinternetguide.com

www.oac.sa.edu.au

www.OxfordSecondary.co.uk

www.sacsa.sa.edu.au

www.scimas.sa.edu.au

www.ssabsa.sa.edu.au/maths

Glossary for Advanced Mathematics

Algebra	Algebra is that part of mathematics that deals with properties and relations of numbers and sets, using symbols to represent unknown quantities
Algorithm	An algorithm is a given series of steps for reaching a solution to a mathematical problem
Alternate angles	Alternate angles are a pair of equal angles formed by a straight line crossing a pair of parallel lines. The alternate angles are between the parallel lines, one on either side of the transversal
Arithmetic	Arithmetic is the study of numbers, arithmos from Greek. It involves computations with whole numbers, fractions and decimals using the operations of addition, subtraction, multiplication and division
Associative	An operation is called associative if it does not matter how the quantities are grouped when they are combined. Both addition and multiplication are associative
Average	Average is a single measure that tries to show the 'middle' of a group of numbers
Bar graph	A bar graph is a graph that uses bars to represent information. The height or the length of the bars is scaled to show size or quantity Another name for bar graph is bar chart
Bearing	A bearing is the angle on the ground measured from the north or south direction that fixes the direction of an object
Box plot	A box plot is a summary graph. The box represents 50% of the data. The lines at each end of the box represent the top 25% of the data
Capacity	Capacity is the amount a container can hold, and refers to a measure of things that can be poured. The standard metric measures of capacity are the millimetre (mL), litre (L), cubic centimetre (cm3) and cubic metre (m3)
Cardinal compass points	The cardinal compass points are the four key directions on a compass: north, south , east and west
Chord	A chord is a straight line that joins two points on a circle. The chord divides the region inside a circle into two segments. A chord that passes through the centre of a circle is called a diameter. Diameters are the longest chord
Compound interest	The calculation of the new amount A when the original amount (the principal), P, of money is subjected to being calculated on interest at the end of a period according to the following formula: $A = P(1+i)^{n.}$
Cummutative	The property of an operation which allows for the order of the values operated with to be interchanged. Addition of real numbers and multiplication are commutative
Cumulative frequency	For data that has been ordered (from minimum to maximum values) the successive values can be assigned freduencies. The cumulative frequency for a value x is the total count of all the data values that are less than or equal in value to x
Experiment	A repeatable activity or process for which each repetition gives rise to exactly one outcome drawn from the sample space (statistical experiment); for example, the observed face of a die. The number of trials observed in the sample size n

Function	A relationship between two sets of variables such that each element of the one set (the domain) is associated with a unique element of the second set (the range)
Global positioning system	A system using satellite and electronic technology, whereby a particular location on the earth's surface is determined in terms of its latitude and longitude
Independent events	The idea that two events do not connect with each other in any observable pattern, and hence that neither event can give any useful information about the other event
Mortgage bond	A loan from a bank, usually for the purchase of property. The loan is subject to the payment of compound interest and is paid off in regular instalments which include interest and capital
Outcome	The result of an experiment (in statistics); for example, the outcome of an experiment in which a die is rolled can be any one of the natural numbers 1 through to 6
Percentiles	Values of ranked data separated into one hundred groups of equal size, especially when sample size n is very large
Probability	For equally likely outcomes, the number of favourable outcomes divided by the total number of possible outcomes of an experiment
Quartiles	Three values which split the ordered sample calues into four groups of equal size. The second quartile is the median
Relative frequency	(of a particular outcome in a statistical experiment) is the number of occurrences of a particular outcome divided by the number of trials
Sample	In statistics, a group of data chosen from all the possible data
Significant figures	The number of digits in a number, starting from the first non-zero digit and including final zeros: for example, 0.0012340 has 5 significant figures
Tessellation	A covering of the plane with shapes, often polygons and in a repetitive manner

Glossary for assessment

Syllabus outcomes, criteria and performance standards, and examination questions all have key words that state what students are expected to be able to do. A glossary of key words has been developed to help provide a common language and consistent meaning in the syllabus and teacher guide documents.

Using the glossary will help teachers and students understand what is expected in response to examinations and assessment tasks.

Glossary of key words for assessment

Account	Account for: state reasons for, report on. Give an account of: narrate a series of events or transactions
Analyse	Identify components and the relationship between them; draw out and relate implications
Apply	Use, utilise, employ in a particular situation
Appreciate	Make a judgement about the value of
Assess	Make a judgement of value, quality, outcomes, results or size
Calculate	Ascertain or determine from given facts, figures or information
Clarify	Make clear or plain
Classify	Arrange or include in classes or categories
Compare	Show how things are similar or different
Construct	Make; build; put together (items or arguments)
Contrast	Show how things are different or opposite
Critically (analyse, evaluate)	Add a degree or level of accuracy, depth, knowledge and understanding, logic, questioning, reflection and quality to (analysis or evaluation)
Deduce	Draw conclusions
Define	State meaning and identify essential qualities
Demonstrate	Show by example
Describe	Provide characteristics and features
Discuss	Identify issues and provide points for and/or against
Distinguish	Recognise or note or indicate as being distinct or different from; to note differences between
Evaluate	Make a judgement based on criteria; determine the value of
Examine	Inquire into
Explain	Relate cause and effect; make the relationships between things evident; provide why and/or how
Extract	Choose relevant and/or appropriate details
Extrapolate	Infer from what is known
Identify	Recognise and name
Interpret	Draw meaning from
Investigate	Plan, inquire into and draw conclusions about

Justify	Support an argument or conclusion
Outline	Sketch in general terms; indicate the main features of
Predict	Suggest what may happen based on available information
Propose	Put forward (for example, a point of view, idea, argument, suggestion) for consideration or action
Recall	Present remembered ideas, facts or experiences
Recommend	Provide reasons in favour
Recount	Retell a series of events
Summarise	Express, concisely, the relevant details
Synthesise	Putting together various elements to make a whole

Appendix A: Formula sheet

Data analysis

standardised score: $z = \frac{x - \bar{x}}{s_x}$

least squares line: $y = a + bx$ where $b = r\frac{s_y}{s_x}$ and $a = \bar{y} - b\bar{x}$

residual value: residual value = actual value – predicted value

seasonal index: seasonal index = $\frac{\text{actual figure}}{\text{deseasonalised figure}}$

Number patterns

arithmetic series: $a + (a + d) + \ldots + (a + (n-1)d) = \frac{n}{2}\left[2a + (n-1)d\right] = \frac{n}{2}(a + l)$

geometric series: $a + ar + ar^2 + \ldots + ar^{n-1} = \frac{a(1 - r^n)}{1 - r}, r \neq 1$

infinite geometric series: $a + ar + ar^2 + ar^3 + \ldots = \frac{a}{1 - r}, |r| < 1$

Geometry and trigonometry

area of a triangle: $\frac{1}{2}bc\sin A$

Heron's formula: $A = \sqrt{s(s-a)(s-b)(s-c)}$ where $s = \frac{1}{2}(a + b + c)$

circumference of a circle: $2\pi r$

area of a circle: πr^2

volume of a sphere: $\frac{4}{3}\pi r^3$

surface area of a sphere: $4\pi r^2$

volume of a cone: $\frac{1}{3}\pi r^2 h$

volume of a cylinder: $\pi r^2 h$

volume of a prism: area of base × height

volume of a pyramid: $\frac{1}{3}$ area of base × height